Summary

In the October 9th meeting, I began by presenting a simple nonlinear example involving the unit circle. I then explained Nagumo’s theorem using this example, illustrating how the invariant set is the unit circle. In this theorem, there are two key conditions: if the dynamic vector is inside the invariant set, it will remain inside for a specific time interval, and if it is on the boundary, there is a condition ensuring that the dynamic vector points inward.

I also presented other theorems, such as Bony's and Brezis' theorems. I explained that there are two vectors, and their dot product must be zero or negative. This condition is then connected to the value function. In these theorems, the dot product involves a normal vector corresponding to the boundary, while the other vector represents the dynamic vector. I mentioned that an input parameter influences the dynamic vector, and since we are considering a maximization problem, we need a set of input parameters that direct the vector inward. I referenced the paper “Viability Theory: An Applied Mathematics Tool for Achieving Dynamic Systems’ Sustainability.”

Overall, I discussed how the value function relates to the normal vector: the zero level set of the value function is the invariant set, and the gradient represents the normal vector to the boundary. To determine the value function, we must solve the HJB equation. However, I raised some questions, such as why the value function is treated as a function of time, input, and states. Dr. Yoon and Dr. Begum explained that the value function is evaluated with initial conditions, but I am unsure if this is correct or why the value function is considered a signed distance function. I understand that a signed distance function is Lipschitz continuous, but there must be a reason why the value function is assumed to be a signed distance function. It seems that the connection between these theorems and the HJB equation is clearer to me now.

After I presented my topics, Dr. Yoon stated that the nature of safety is a differential game. He also mentioned that MPC cannot handle scenarios where the second player is not exclusively an external disturbance. We ultimately decided to focus on the Hamilton-Jacobi-Isaacs (HJI) formulation. I also asked Dr. Begum to send me the references she mentioned. I agreed to revisit the general safety framework paper and review it for our next meeting on Wednesday.